Computing Cournot Equilibria in Two Settlement Electricity Markets with Transmission Constraints[1]

Jian Yao[2], Shmuel S. Oren[2], Ilan Adler[2]

Department of Industrial Engineering and Operations Research
4135 Etcheverry Hall,
University of California at Berkeley, Berkeley CA 94720
{jyao, oren, adler}@ieor.berkeley.edu

***Abstract*— We formulate a two-settlement equilibrium in competitive electricity markets as a subgame-perfect Nash equilibrium in which each generation firm solves a Mathematical Program with Equilibrium Constraints (MPEC), given other firms' forward and spot strategies. We implement two computational approaches, one of which is based on a Penalty Interior Point Algorithm and the other is based on a steepest descent approach. We apply the algorithm to a six node illustrative example.**

I. Introduction

The last decade has witnessed a fundamental transformation of the electric power industry around the world from one dominated by regulated vertically integrated monopolies to an industry where electricity is produced and traded as a commodity through competitive markets. In the US, this transformation was pioneered in the late 1990s by California and the northeastern power pools including Pennsylvania-New Jersey-Maryland (PJM) Interchange, New York and New England, which established markets for electricity. A recent arrival is the ERCOT market in Texas. Lessons from the accumulated experience in the early-restructured markets are being incorporated in market reforms and new market designs. While there are significant differences among the many implemented and proposed market designs that vary in terms of ownership structure, level of centralization and the authority of the system operator, most market designs in the US have adopted or are in the process of adopting a multi settlement system approach where forward transactions, day ahead transactions and real time balancing transactions are settled at different prices. Theoretical analysis and empirical evidence suggests that forward trading reduces the incentives of sellers to manipulate spot market prices by reducing the sensitivity of sellers' profits to spot prices fluctuations. Thus forward trading is viewed as an effective way of mitigating market power at real time. However, due to the complexity of the problem, it is not clear to what extent suppliers are willing to engage in forward transactions. Furthermore it is not well understood whether forward trading may in fact help generators with market power in the spot market to lock in or even increase Oligopoly rents.

In this paper, we formulate the two-settlement competitive electricity market as a two-period game, and its equilibrium as a subgame-perfect Nash equilibrium (see [8]) expressed in the format of an Equilibrium Problem with Equilibrium Constraints, in which each firm faces a Mathematical Programming problem with (linear) Equilibrium Constraints (MPEC) given other firms' commitments in forward contracts. We implement two solution approaches which are based on Penalty Interior Point Algorithm (PIPA) and Steepest Descent Method and apply them to a 6-node and 8-line illustrative example. For the specific data and simplifying assumptions of the example, both approaches give the same result showing that in the equilibrium, firms commit certain quantities in forward transactions and adjust their positions in the spot market responding to contingencies and demand realization. We plan to explore this issue in future work under the more realistic assumption of quadratic cost functions.

Since our model relies on solving Complementarity Problems and Mathematical Program with Equilibrium Constraints (MPECs), we shall first introduce these concepts. A Mixed Complementarity Problem (MCP) is defined as follows: find vector $x \in R^n$ such that $x \geq 0, f(x) \geq 0, f(x)^T x = 0$ and $g(x) = 0$, where functions $f : R^n \to R^n$ and $g : R^n \to R^m$ are given. If $f(x)$ and $g(x)$ are affine functions, the MCP is a mixed Linear Complementarity Problem (mixed LCP). If $g(x)$ is omitted, it becomes a Linear Complementarity Problem (LCP) (see [7]).

A MPEC (see [15]) is an optimization problem with two sets of variables, x and y, in which some or all of its constraints are defined by a parametric variational inequality (sometimes called complementarity system) with y as its primary variables and x as the parameter vector. Specifically, suppose that $f : R^{n+m} \to R$ and $F : R^{n+m} \to R^m$ are given functions, $Z \subseteq R^{n+m}$ is a non-empty closed set, and $C : R^n \to R^m$ is a set-valued map with (possibly empty) closed convex values.

The MPEC is defined as:

$\min f(x, y)$
subject to:
$(x, y) \in Z$ and $y \in S(x)$

where $S(x)$ is the solution set of the variational inequality

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defined by the pair $(F(x), C(x))$; i.e. $y \in S(x)$ if and only if $y \in C(x)$ and $(v-y)^T F(x,y) \geq 0$ for all $v \in C(x)$.

The rest of this paper is arranged as follows. Related research on models with transmission constraints and forward markets are reviewed in the following section, section III presents the model assumptions and the mathematical formulation. Two approaches for solving the problems are given in section IV. An example, numerical results and conclusions are presented in sections V and VI.

II. Related Research

We review models of spot energy markets with transmission constraints as well as models that include forward markets. Most of the spot market models with transmission constraints assume either perfect competition or an oligopoly based on the Cournot conjectural variation. Assuming that the agents act as price takers in the transmission market allows such models to be solved as complementarity problems or variational inequalities.

Wei and Smeers [20] consider a Cournot model with regulated transmission prices. They solve the variational inequalities to determine unique long-run equilibria in their models. In subsequent work, Smeers and Wei [19] consider a separated energy and transmission market, where the system operator conducts a transmission capacity auction with power marketers purchasing transmission contracts to support bilateral transactions. They conclude that such market converges to the optimal dispatch for a large number of marketers. Borenstein and Bushnell [3] use a grid search algorithm that iteratively converges to a Cournot model based on data from the California market.

Hobbs et al [12] calculate a Cournot equilibrium under the assumptions of linear demand and cost functions, which leads to a linear mixed complementarity problem. In a market without arbitrageurs, non-cost based price differences can arise because the bilateral nature of the transactions gives firms more degrees of freedom to discriminate between electricity demand at various nodes. This is equivalent to a separated market as in [19]. In the market with arbitrageurs, any non-cost differences are arbitraged by trades who buy and sell electricity at nodal prices. This equilibrium is shown to be equivalent to a Nash-Cournot equilibrium in a POOLCO-type market. In another paper [13], Hobbs presents an oligopolistic market where each firm submits a linear supply function to the Independent System Operator (ISO). He assumes that firms can only manipulate the intercepts of the supply functions, but not the slopes, while power flows and pricing strategies are constrained by the ISO's linearized DC optimal power flow. Each firm in this model faces an MPEC problem with spatial price equilibrium as the inner problem.

Work in forward markets has focused on the welfare enhancing properties of forward markets and the commitment value of forward contracts. The basic model in Allaz [1] assumes that producers meet in a two period market where there is some demand uncertainty in the second period. Allaz shows that generators have a strategic incentive to contract forward if other producers do not. This result can be understood using the concepts of strategic substitutes and complements of Bulow, Geneakoplos and Klemperer [4]. In these terms, the availability of the forward market makes a particular producer more aggressive in the spot market. Due to the strategic substitutes effect, this produces a negative effect on its competitors' production. The producer with access to the forward market can therefore use its forward commitment to improve its profitability to the detriment of its competitors. Allaz shows, however, that if all producers have access to the forward market, it lead to a prisoners' dilemma type of effect, reducing profits for all producers. Allaz and Vila [2] extend this result to the case where there is more than one time period where forward trading takes place. For a case without uncertainty, they establish that as the number of periods when forward trading takes place tends to infinity, producers lose their ability to raise market prices above marginal cost and the outcome thanks to the competitive solution.

von der Fehr and Harbord [9] and Powell [18] study contracts and their impact on an imperfectly competitive electricity spot market: the UK pool. von der Fehr and Harbord [9] focus on price competition in the spot market with capacity constraints and multiple demand scenarios. They find that contracts tend to put downward pressure on spot prices. Although, this provides disincentive to generators to offer such contracts, there is a countervailing force in that selling a large number of contracts commits a firm to be more aggressive in the spot market, and ensures that it is dispatched into its full capacity in more demand scenarios. Powell [18] explicitly models recontracting by Regional Electricity Companies (RECs.) after the maturation of the initial portfolio of contracts set up after deregulation. He adds risk aversion on the part of RECs to earlier models. Generators act as price setters in the contract market. He shows that the degree of coordination has an impact of the hedge cover demanded by the RECs, and points to a "free rider" problem which leads to a lower hedge cover chosen by the RECs.

Newbery [16] analyzes the role of contracts as a barrier to entry in the England and Wales electricity market. He extends earlier work by modeling equilibria of supply functions in the spot market. He further shows that if entrants can sign base load contracts and incumbents have enough capacity, the incumbent can sell enough contacts to drive down the spot price below the entry deterring level, resulting in more volatile spot prices if producers coordinate on the highest profit SFE. Capacity limit however may imply that incumbents cannot play a low enough SFE in the spot market and hence cannot deter entry. Green [11] extends Newbery's model showing that when generators compete in SFEs in the spot market, together with the assumption of Cournot conjectural variations in the forward market, imply that no contracting will take place unless buyers are risk averse and willing to provide a hedge premium in the forward market. He shows that forward sales can deter excess entry, and increase economic efficiency and long-run profits of a large incumbent firm faced with potential entrants.

III. The model

A. Introduction and assumptions

We introduce a model for calculating the equilibrium quantities and prices of electricity over a given network for a given period that is denoted as period 2. We consider two markets, the spot market that operates at period 2, and a forward market that operates at a preceding period that we denote as period 1.

We model the spot market in the topology of the transmission network through the DC approximation of Kirchoff's laws. Specifically, flows on lines can be calculated by a power transfer distribution factor (PTDF) which gives the proportion of flow on a particular line resulting from an injection of one unit at a particular node and a corresponding withdrawal at an arbitrary (but fixed) "slack bus"[6]. Different PTDF matrices with corresponding probabilities characterize uncertainty regarding the realized network topology in the spot market. To avoid non-convex issue in the spot market (see [5], [17]), we assume that agents do not game the transmission prices. For simplicity we assume that all nodes are both demand nodes and generation nodes and that there is exactly one firm owning generation facility at each node. The capacities of the facilities (as well as of the lines) are unknown at period 1 and are subject to stochastic fluctuations in period 2. To further simplify the formulation we assume no wheeling fees.

Firms enter contracts in the forward market (which is organized in zones that may include several nodes) in period 1, which are settled financially in period 2, based on a weighted average of the nodal prices corresponding to the nodes in each respective zone. The weights are typically based on historical load ratio for each node. We assume that risk neutral speculators take opposite positions to the firms and that by anticipating any arbitrage opportunities the forward price in a zone is set equal to the expected weighted nodal prices (of the same zone) from the spot market. We plan to relax this assumption in future work to capture how lack of liquidity (or high risk aversion) on the buyers side might be reflected in a high risk premium embedded in the forward prices.

We view the two settlement in the electricity market as a complete information game with two periods. Our formulation approach is to model the equilibrium in this two period model as a subgame-perfect Nash equilibrium. We model the second period of the game as a subgame with three stages. In the first stage Nature determines the state of the world (and thus settles the actual capacities of the generation facilities and the transmission lines as well as the shape of the demand and cost functions at each node). In the second stage, firms anticipate arbitrage in stage three and compete in a Nash-Cournot manner. In the third stage, the system operator arbitrages any non-cost differences in nodal energy prices so that there is no spatial discrimination in energy prices subject to transmission congestion.

Specifically, the equilibrium is determined by considering three classes of optimization problems:

Problem $\mathcal{G}_g$: generation firms' decision problems in the second stage of the spot market.

Problem $\mathcal{S}$: The system operator decision problem in the third stage of the spot market.

Problem $\mathcal{F}_g$: The generation firms' decision problems in the forward market in which the preceding problems are imbedded and whose solutions provide the equilibrium entities.

B. Notation

(1) Sets:

- N: The set of all nodes
- Z: The set of all zones
- L: The set of all transmission lines
- C: The set of all states of contingencies
- G: The set of all generation firms
- N_g^G: The set of nodes at which firm g owns generating facilities.
- N_z^Z: The set of nodes in zone z. We denote by
- $z(i)$ The zone where node i resides.

(2) Decision variables:

The variables related to the forward markets are:

- f_z: The forward price in zone z.
- $x_{g,z}$: The forward quantity of firm g in zone z.

The variables related to the spot markets are:

- r_i^c : Adjustment quantity into/from node i by the system operator in state c.
- q_i^c: The quantity generated at node i in state c.

(3) Parameters:

- $\overline{q}_i^c$: Capacity of generation facility at node i in state c.
- $p_i^c(\cdot)$: Inverse demand function at node i in state c. We assume that the inverse demand function is linear where $p_i^c(q) = a_i^c - b_i^c q$,
- $s_i(\cdot)$: Cost function at node i. We assume that the cost function is linear where $s_i(q) = d_i q$.
- K_l^c : capacity limit of line l in state c.
- $D_{l,i}^c$: Power transfer distribution factor in state c on line l with respect to node i.
- $\overline{x}_{g,z}$: An upper bound on the forward quantity $x_{g,z}$, which we assume to be the total capacity of firm g's facilities in zone z at the "normal" state.
- ρ_g: Firm g's risk-aversion coefficient.
- $prob(c)$: Probability of state c in the spot market.

C. The Formulation

The no-arbitrage assumption implies that the forward price is equal to the expected value of weighted spot nodal prices. That is:

$$f_z = E[\sum_{i \in N_z^Z} \delta_i p_i^c(q_i^c + r_i^c)] \qquad \text{(p1)}$$

where δ_i $(\delta_i \geq 0\ ,\ \sum_{i \in N_z^Z} \delta_i = 1)$ are weights that are used to settle the forward contracts.

In stage two of the second period, for a given state c, each firm g solves the following profit maximization problem :

$$\mathcal{G}_g :\ \max_{q_i^c}\ \pi_g^c$$

subject to:

$$0 \leq q_i^c \leq \overline{q}_i^c \qquad i \in N_g^G \qquad \text{(g1)}$$

where $\pi_g^c = \sum_{i\in N_g^G}[p_i^c(q_i^c+r_i^c)q_i^c - s_i(q_i^c)] + \sum_{z\in Z}(f_z - \sum_{i\in N_z^Z}\delta_i p_i^c(q_i^c+r_i^c))x_{g,z}$ is the profit of firm g in state c in the spot market.

Following the preceding problems, the system operator solves the following social welfare maximization problem:

$$\mathcal{S}: \max_{r_i^c} \sum_{i\in N}[\int_0^{r_i^c} p_i^c(q_i^c+w_i)dw_i]$$

subject to:

$$\sum_{i\in N} r_i^c = 0 \qquad \text{(s1)}$$

$$-K_l^c \le \sum_{i\in N} D_{l,i}^c r_i^c \le K_l^c, \quad l\in L \qquad \text{(s2)}$$

$$q_i^c + r_i^c \ge 0, \quad i\in N \qquad \text{(s3)}$$

To rational for the system operator's problem, is that in the absence of wheeling fees, it is possible to gain social surplus by output/input dw_i units of electricity from/to node i while input/output it to/from other nodes until the prices at the nodes are equal, or until some transmission lines are saturated.

Since the nodal inverse demand functions as well as the cost functions are assumed to be linear, both problem $\mathcal{G}_g$ and $\mathcal{S}$ are concave quadratic programming problems, which implies that first order necessary conditions (the KKT conditions) are also sufficient. Thus, we can replace problems $\mathcal{G}_g$ and $\mathcal{S}$ by their KKT conditions.

Let α^c be the Lagrangian multiplier to constraint (s1), λ_{l-}^c and λ_{l+}^c be the Lagrangian multipliers to constraint (s2), and β_i^c be the Lagrangian multipliers to constraint (s3). Then the KKT conditions for problem $\mathcal{S}$ (including the feasibility constraints) are:

for $c\in C$, $l\in L$, $i\in N$

$$\sum_{j\in N} r_j^c = 0 \qquad \text{(KKT1)}$$

$$a_i^c - b_i^c(q_i^c+r_i^c) - \alpha^c + \beta_i^c + \sum_{t\in L}(\lambda_{t-}^c D_{t,i}^c - \lambda_{t+}^c D_{t,i}^c) = 0 \qquad \text{(KKT2)}$$

$$\lambda_{l-}^c \ge 0 \qquad \text{(KKT3)}$$

$$\sum_{j\in N} D_{l,i}^c r_j^c + K_l^c \ge 0 \qquad \text{(KKT4)}$$

$$\lambda_{l-}^c(\sum_{i\in N} D_{l,i}^c r_i^c + K_l^c) = 0 \qquad \text{(KKT5)}$$

$$\lambda_{l+}^c \ge 0 \qquad \text{(KKT6)}$$

$$K_l^c - \sum_{j\in N} D_{l,j}^c r_j^c \ge 0 \qquad \text{(KKT7)}$$

$$\lambda_{l+}^c(K_l^c - \sum_{j\in N} D_{l,j}^c r_j^c) = 0 \qquad \text{(KKT8)}$$

$$\beta_i^c \ge 0 \qquad \text{(KKT9)}$$

$$q_i^c + r_i^c \ge 0 \qquad \text{(KKT10)}$$

$$\beta_i^c(q_i^c + r_i^c) = 0 \qquad \text{(KKT11)}$$

Similarly, let η_i^c and γ_i^c be the Lagrangian multipliers associated with constraint (g1), then the KKT conditions for problem $\mathcal{G}_g$ are:

for $c\in C$, $i\in N$

$$a_i^c - 2b_i^c q_i^c - b_i^c r_i^c - d_i + (1-prob(c))\delta_i b_i^c x_{g,z(i)} - \gamma_i^c + \eta_i^c = 0 \qquad \text{(KKT12)}$$

$$\gamma_i^c \ge 0 \qquad \text{(KKT13)}$$

$$\overline{q}_i^c - q_i^c \ge 0 \qquad \text{(KKT14)}$$

$$\gamma_i^c(\overline{q}_i^c - q_i^c) = 0 \qquad \text{(KKT15)}$$

$$\eta_i^c \ge 0 \qquad \text{(KKT16)}$$

$$q_i^c \ge 0 \qquad \text{(KKT17)}$$

$$\eta_i^c q_i^c = 0 \qquad \text{(KKT18)}$$

In period 1, each firm g determines the forward quantities (bounded by the capacities of the facilities of firm g) by maximizing the value of the forward transactions subject to the KKT conditions (KKT1-KKT18) which represent the anticipated actions in period 2. Thus firm g optimization problem in period 1 is:

$$\mathcal{F}: \max_{x_{g,z}} E[\pi_g] - \frac{\rho_g}{2}var(\pi_g)$$

subject to:

$$x_{g,z} \le \overline{x}_{g,z} \quad z\in Z \qquad \text{(f1)}$$

and constraints (p1), (KKT1-KKT18)

D. Further Conversion

Problem $\mathcal{F}$ is a Mathematical Program with Equilibrium Constraints (MPEC). For the purpose of further simplification, we define

- x_g: The vector of firm g's forward variables. $x_g = [x_{g,z}, z\in Z]$
- x: The vector of all firms' forward variables. $x = [x_g, g\in G]$
- y: The vector of lagrangian multipliers for all inequality constraints.
$$y = \left[\begin{array}{c|c} \eta_i^c & \\ \gamma_i^c & \\ \beta_i^c & c\in C, i\in N, l\in L \\ \lambda_{l-}^c & \\ \lambda_{l+}^c & \end{array}\right]$$
- v: The vector of adjustment quantities r_i^c and the multipliers α^c.
$$v = \left[\begin{array}{c|c} r_i^c & \\ \alpha^c & c\in C, i\in N \end{array}\right]$$
- w: The slackness of the inequality constraints.
$$w = \left[\begin{array}{c|c} q_i^c & \\ \overline{q}_i^c - q_i^c & \\ q_i^c + r_i^c & c\in C, i\in N, l\in L \\ \sum_{j\in N} D_{l,j}^c r_j^c + K_l^c & \\ K_l^c - \sum_{j\in N} D_{l,j}^c r_j^c & \end{array}\right] \qquad \text{(w1)}$$

Then constraints (KKT1-KKT18) and (w1) become a mixed LCP with respect to w, y and v with x being the parameter. Note that v can be solved from constraints (KKT1) and (KKT2), we can eliminate v from this mixed LCP, to obtain:

$$w = a + Ax + My$$
$$w \ge 0, y \ge 0$$
$$w^T y = 0$$

where a, A, and M are suitable vector and matrices derived from (KKT1-KKT18) and (w1).

Thus the two-settlement equilibrium can be converted to an Equilibrium Problem with Equilibrium Constraints (EPEC), in which each firm faces (given other firms' commitments) an MPEC problem:

$$\min_{x_g,y,w} f(x,y,w)$$

subject to:

$$x_{g,z} \le \overline{x}_{g,z} \ \forall z\in Z \qquad \text{(f1)}$$

$$w = a + Ax + My \qquad \text{(EC1)}$$

$$w \ge 0, y \ge 0 \qquad \text{(EC2)}$$

$$w^T y = 0 \quad \text{(EC3)}$$

where $f(x, y, w)$ is the objective function of problem $\mathcal{F}$ expressed in term of x, y, and w.

Theorem 1: If a_i^c are the same for all $i \in N$ for any given state c, and they are greater than d_i, then the left-hand inequality of constraint (g1) and costraint (s3) are never binding in the optimal solution.

Proof: This is not hard to see: In problem $\mathcal{G}_g$, generating some small quantity at any node i always dominates generating nothing; and in problem $\mathcal{S}$, dispatching all generated quantity on any node to other nodes will never maximize social welfare. ■

Theorem 2: The conditions of Theorem 1 implies that M is symmetric and positive semi-definite.

Proof: Note that the left-hand inequality of (g1) and constraint (s3) are never binding in the optimal solution by the assumption of theorem 1. Thus, we can drop these two constraints, as well as the corresponding multiplies, from the LCP problem.

Also note that the LCP problem above can be divided into sub-problems according to the states of contingencies, therefore it suffices to prove M is symmetric and positive semi-definite for each state.

For any state c, let $e \in R^{|N|}$ be a vector with all ones, B be a diagonal matrix with $b_{ii} = b_i^c, \forall i \in N$, and D be a matrix with $D_{l,i} = D_{l,i}^c, \forall l \in L, \forall i \in N$. It can be shown that

$$M = \begin{bmatrix} 0 & 0 & 0 \\ 0 & DQD^T & -DQD^T \\ 0 & -DQD^T & DQD^T \end{bmatrix} + \begin{bmatrix} H^{-1} & H^{-1}BQD^T & -H^{-1}BQD^T \\ DQBH^{-1} & DQBH^{-1}BQD^T & -DQBH^{-1}BQD^T \\ -DQBH^{-1} & -DQBH^{-1}BQD^T & DQBH^{-1}BQD^T \end{bmatrix}$$

where $Q = B^{-1} - \frac{B^{-1}ee^TB^{-1}}{e^TB^{-1}e}$ is symmetric positive semi-definite and $H = -BQB + 2B$ is symmetric positive definite.

Moreover, note that M can be written as:

$$M = \begin{bmatrix} 0 \\ (DQD^T)^{\frac{1}{2}} \\ -(DQD^T)^{\frac{1}{2}} \end{bmatrix} \begin{bmatrix} 0 & (DQD^T)^{\frac{1}{2}} & -(DQD^T)^{\frac{1}{2}} \end{bmatrix} + \begin{bmatrix} H^{-\frac{1}{2}} \\ DQBH^{-\frac{1}{2}} \\ -DQBH^{-\frac{1}{2}} \end{bmatrix} \begin{bmatrix} H^{-\frac{1}{2}} & H^{-\frac{1}{2}}BQD^T & -H^{-\frac{1}{2}}BQD^T \end{bmatrix}$$

Therefore, M must be symmetric positive semi-definite. ■

Following the monograph [7], the LCP problem (EC1-EC3) satisfies the w-uniqueness condition, thus it also has a unique solution given that problems $\mathcal{S}$ and $\mathcal{G}_g$ are both non-degenerate.

IV. Solution Approaches

The EPEC in the forward market are constrained in non-convex regions by the LCP problem (EC1-EC3), therefore we cannot write down the optimality conditions for each agent and aggregate them into a large problem which we can solve directly. We attempt to solve for an equilibrium, if at least one exists, by iteratively deletion of dominated strategies, that is, we sequentially solve each firm's optimization problem using as data optimal value from previously solved problems. Thus, starting from a feasible vector of forward variables x, we solve for x_1 using x_{-1} as data in the first firm's optimization problem, where "x_{-1}" means all firms' forward variables except for firm 1's; then use x_{-2} to solve x_2, and so on. In each iteration, we solve a MPEC problem for one firm. Based on the technologies solving MPEC problems, we develop two approaches, namely iterative Response Surface Method (RSM) and iterative Penalty Interior Point Algorithm (PIPA).

A. Approach 1: iterative RSM

In this approach, we solve MPEC problems using Response Surface Method (RSM) by iteratively searching steepest descent (SD) direction. For some small positive value ϵ, called step size, we define x_g's neighborhood set as all feasible points with the format $(x_{g,z} \pm \epsilon, \forall z \in Z)$. When searching steep descent direction, we test the responding objective value for each point in the neighborhood set of x_g combined with x_{-g}, by solving w and y through PATH solver [8], and the only point with best objective value is kept for next search. Figure 1 illustrates the case when $|Z| = 2$.

Fig. 1. Neighborhood set and SD direction

The following describes this approach:

1. (Initiation) Select initial values of x^0 and ϵ_0. $k := 0$.
2. (Main loop) Let $x^{k+1} := x^k$, increase k by 1, let $g := 1$
3. (Steepest descent direction search) Let s^* be the point with highest objective value in the neighborhood set of x_g^k. If s^* has a worse objective value than x_g^k, go to step 4; else let $x_g^k := s^*$ and repeat this step.
4. (Next firm) Let $x_g^k = 0.5(x_g^k + x_g^{k-1})$. If $g < |G|$, increase g by 1 and go to step 3.
5. (Termination check) If ($x^k = x^{k-1}$) and (ϵ_k is small enough), stop and a solution is found; otherwise, set $\epsilon_{k+1} := \epsilon_k/2$, then go to step 2.

B. Approach 2: iterative PIPA

In this approach, we solve MPEC problems using Penalty Interior Point algorithm (PIPA). See the monograph [15] for more details regarding PIPA. Our key idea of penalty interior point method is as follows. Assume we have a MPEC problem:

$$\begin{aligned} &\min f(x, y, w, v) \\ &\text{subject to :} \\ &x \in X \\ &F(x, y, w, v) = 0 \end{aligned}$$

$y \geq 0, w \geq 0$
$y^T w = 0$

where $x \in R^n, y \in R^m, w \in R^m, v \in R^l$. We define two auxiliary functions:
the constraints violation function

$$\psi(x,y,w,v) = F(x,y,w,v)^T F(x,y,w,v) + w^T y,$$

and the penalized objective function

$$P_\alpha(x,y,w,v) = f(x,y,w,v) + \alpha\psi(x,y,w,v)$$

Given a point $(x,y,w,v) \in X \times R^m_{++} \times R^m_{++} \times R^l$, we solve a quadratic program whose solution yields a descent direction for the function P_α. A one-dimension search is carried out along this direction so as to decrease P_α. This new (x,y,w,v) is then the starting point for the next search.

The PIPA algorithm solves above MPEC problem in the following steps.

0. (Initiation) let $(x^0, y^0, w^0, v^0) \in X \times R^m_{++} \times R^m_{++} \times R^l$ be given. Let $\overline{\rho}$ satisfying
$\overline{\rho}\frac{(y^0)^T w^0}{m} \leq \min\limits_{1\leq i\leq m} y^0_i w^0_i$
and $\sigma_0 \leq \min(\overline{\rho}, 0.1)$. Set $r := 0, \alpha_0 := 1.2$.

1. (Direction generation) let the unique optimal solution of the quadratic programming be (dx^r, dy^r, dw^r, dv^r):

$\min(df^r_x)dx + (df^r_y)dy + (df^r_w)dw + (df^r_v)dv + 0.5(dx)^T dx$
subject to:
$x^r + dx \in X$
$-\sqrt{\| F^r \|^2_2 + (w^r)^T y^r} e \leq dx \leq \sqrt{\| F^r \|^2_2 + (w^r)^T y^r} e$
$(dF^r_x)dx + (dF^r_y)dy + (dF^r_w)dw + (dF^r_v)dv = -F^r$
$diag(w^r)dy + diag(y^r)dw = -diag(w^r)y^r + \sigma_r \frac{(y^r)^T w^r}{m} e$
where $F^r = F(x^r, y^r, w^r, v^r)$.

Let $\alpha_r := \alpha^p_{r-1}$ where p is the smallest integer such that
$(df^r_x)dx^r + (df^r_y)dy^r + (df^r_w)dw^r + (df^r_v)dv^r$
$-\alpha^p_{r-1}(2 \| F^r \|^2_2 + (1-\sigma_r)(y^r)^T w^r)$
$\leq \psi(x^r, y^r, w^r, v^r)$

2. (Step size determination) define a linear function
$g_r(\tau) = (1-\overline{\rho})\sigma_r \frac{(y^r)^T w^r}{m} + \tau(\min\limits_{1\leq i\leq m} dy^r_i dw^r_i - \overline{\rho}\frac{(dy_r)^T dw^r}{m})$
Let $\overline{\tau_r}$ be the unique root of the function $g_r(\tau)$ for $\tau \in (0,1)$ if this root exists; let $\overline{\tau_r} := 0.95$ if $g_r(\tau)$ has no root in (0,1]. Let $\tau_r := \overline{\tau_r} 0.95^k$ where k is the smallest nonnegative integer such that

$x^r(\tau_r) = x^r + \tau_r dx^r$
$y^r(\tau_r) = y^r + \tau_r dy^r$
$w^r(\tau_r) = w^r + \tau_r dw^r$
$v^r(\tau_r) = v^r + \tau_r dv^r$
$\psi(x^r(\tau_r), y^r(\tau_r), w^r(\tau_r), v^r(\tau_r)) \leq \psi(x^r, y^r, w^r, v^r)$
$P_{\alpha_r}(x^r(\tau_r), y^r(\tau_r), w^r(\tau_r), v^r(\tau_r)) - P_{\alpha_r}(x^r, y^r, w^r, v^r)$
$\leq 0.5\tau_r((df^r_x)dx^r + (df^r_y)dy^r + (df^r_w)dw^r + (df^r_v)dv^r)$
$-\alpha_r(2 \| F^r \|^2_2 + (1-\sigma_r)(y^r)^T w^r)$
$\leq -0.5\tau_r \psi(x^r, y^r, w^r, v^r)$

3. (Termination check) Define $x^{r+1} := x^r(\tau_r)$, $y^{r+1} := y^r(\tau_r)$, $w^{r+1} := w^r(\tau_r)$, $v^{r+1} := v^r(\tau_r)$. If stopping rule is satisfied, terminate; otherwise choose σ_{r+1} to be a scalar satisfying $0 < \sigma_{r+1} \leq \sigma_r$ and return to step 1 with r replaced by $r+1$.

The following gives the detail of this approach:

1. (Initiation) Select initial values of x_0, $k := 0$
2. (Main loop) Let $x^{k+1} := x^k$, increase k by 1, let $g := 1$.

Fig. 2. An example

3. (PIPA) Calculate x^k_g while treating x^k_{-g} as constants through PIPA algorithm.
4. (Next firm) Let $x^k_g := 0.5(x^k_g + x^{k-1}_g)$. If $g < |G|$, increase g by 1 and go to step 3.
5. (Termination check) If the error $\| x^{k-1} - x^k \|$ is enough small, stop and a solution is found; otherwise, go to step 2.

As you will see, these approaches succeed with our formulation and the example. However, one has to be careful with these approaches, if they are applied to some general EPEC problems. It might happen that:

1. Neither of these two approaches can guarantee convergence.
2. Neither of these two approaches can only find pure-strategy equilibria. Therefore, they will fail if applied to games with only mixed-strategy equilibria. For example, if these approaches are applied to a zero-sum matching-penny game, they will never converge.
3. Noting that both approaches search for local minimum point, we can only guarantee that the limiting point, if any, is only the firms' locally best responses to one another. To test the limiting point is actually Nash equilibrium, we have to verify that it is the firms' globally best responses to one another. This verification can only be done by grid search. If the verification fails, we need to start over.

V. A NUMERICAL EXAMPLE

In this example, we consider the setup in which the market has two zones (see figure 2). Each zone has three nodes. Zone 1 contains nodes 1, 2 and 3, while nodes 4, 5, and 6 are in zone 2. There are two firms in the market: firm 1 and firm 2. Firm 1 owns generation facilities at node 1, 3 and 4 while the firm 2 owns generation facilities at other three nodes. There are eight transmission lines, each of which has the same electric characteristics except that the flow gates, line 2-4 and line 3-5,

TABLE II
NODAL INFORMATION IN THE NORMAL STATE.

Node	Inverse demand function	weight	marginal cost	capacity	PTDF on line 2-4	PTDF on line 3-5
1	70 -87.5q	1/3	30	4.5	0.5	0.5
2	70 -87.5q	1/3	30	4.5	0.6	0.4
3	70 -87.5q	1/3	25	4.5	0.4	0.6
4	70 -100q	1/3	30	4.5	-0.1	0.1
5	70 -100q	1/3	20	4.5	0.1	-0.1
6	70 -100q	1/3	20	4.5	0	0

TABLE I
STATE OF CONTINGENCIES

state	Probability	Type and Description
1	.82	Normal state: Data is set as in table II.
2	.03	Demand uncertainty: All demands increase by 10%.
3	.03	Demand uncertainty: All demands decrease by 10%.
4	.03	Network uncertainty: Line 2-4 goes down.
5	.03	Network uncertainty: Line 3-5 goes down.
6	.03	Generation uncertainty: Facility at node 4 goes down.
7	.03	Generation uncertainty: Facility at node 2 goes down.

TABLE III
SPOT GENERATION QUANTITIES DUE TO FORWARD

state	node 1	node 2	node 3	node 4	node 5	node 6
1	0.245	0.187	0.304	0.196	0.331	0.333
2	0.436	0.126	0.510	0.286	0.582	0.595
3	0.394	0.085	0.457	0.250	0.526	0.539
4	0.435	0.126	0.504	0.247	0.534	0.546
5	0.435	0.126	0.504	0.247	0.534	0.546
6	0.441	0.131	0.510	0	0.576	0.589
7	0.439	0	0.503	0.262	0.557	0.565

are more likely to be congested. The capacities of line 2-4 and line 3-5 is set to be 0.15.

The uncertainty of the spot market is classified into seven states of contingencies. Six out of seven states are of uncertainty with small independent probabilities, two are of demand uncertainty, two are of network uncertainty and the rest two are of generation uncertainty (see table I). Table II lists the nodal information in the normal state.

We test these two algorithms from a set of different starting points. We also test these algorithms in different firms' orders: in some cases, we solve firm 1's forward commitment followed by solving firm 2's commitment; in other cases, we first find firm 2's forward commitment, then find firm 1's. All of these trials give us the same result, in which the forward commitments are $(x_{1,1}, x_{1,2}, x_{2,1}, x_{2,2}) = (1.1968, 0.7494, 0.2294, 1.3306)$.

Table III through table VI lists the corresponding spot generation quantities, spot prices and adjustment quantities. Compared to the result of single-settlement market, we find

TABLE IV
SPOT PRICES DUE TO FORWARD

state	node 1	node 2	node 3	node 4	node 5	node 6
1	45.18	45.18	45.18	45.18	45.18	45.18
2	34.17	34.17	34.17	34.17	34.17	34.17
3	31.10	31.10	31.10	31.10	31.10	31.10
4	34.57	34.57	34.57	30.76	30.76	30.76
5	34.57	34.57	34.57	30.76	30.76	30.76
6	35.05	35.05	35.05	35.05	35.05	35.05
7	34.89	35.34	34.44	32.18	33.08	32.63

TABLE V
ADJUSTMENT QUANTITIES IN THE SPOT MARKETM DUE TO FORWARD

state	node 1	node 2	node 3	node 4	node 5	node 6
1	0.039	0.097	-.020	0.052	-.083	-.085
2	0.015	0.325	-.060	0.108	-.188	-.201
3	0.006	0.315	-.057	0.100	-.176	-.189
4	-.030	0.279	-.099	0.145	-.141	-.154
5	-.030	0.279	-.099	0.145	-.141	-.154
6	-.041	0.268	-.110	0.350	-.227	-.240
7	-.038	0.396	-.096	0.117	-.188	-.191

TABLE VI
SPOT GENERATION QUANTITIES WITHOUT FORWARD

state	node 1	node 2	node 3	node 4	node 5	node 6
1	0.206	0.206	0.263	0.180	0.280	0.280
2	0.226	0.226	0.289	0.198	0.308	0.308
3	0.185	0.185	0.237	0.162	0.252	0.252
4	0.206	0.206	0.263	0.180	0.280	0.280
5	0.206	0.206	0.263	0.180	0.280	0.280
6	0.223	0.223	0.280	0	0.295	0.295
7	0.226	0	0.283	0.198	0.298	0.298

TABLE VII
SPOT PRICES WITHOUT FORWARD

state	node 1	node 2	node 3	node 4	node 5	node 6
1	48.00	48.00	48.00	48.00	48.00	48.00
2	48.00	48.00	48.00	48.00	48.00	48.00
3	48.00	48.00	48.00	48.00	48.00	48.00
4	48.00	48.00	48.00	48.00	48.00	48.00
5	48.00	48.00	48.00	48.00	48.00	48.00
6	49.52	49.52	49.52	49.52	49.52	49.52
7	49.76	49.76	49.76	49.76	49.76	49.76

that in the two-settlement market

- Lines not congested in the single-settlement market might be congested in the spot market of the two-settlement market. State 4, 5 and 7 in the single-settlement market have no congested lines, therefore the nodal prices are equal; however in the spot market of the two-settlement market, the nodal prices are different due to congestions.
- Not all generation facilities will increase quantities in the spot market of the two-settlement market. The generation facility at node 2 is such an example.
- Consumer surplus increases from 15.4243 in the single-settlement market to 24.0342 in the two-settlement market. Producer surplus decreases from 32.4116 to 30.3323. Social welfare increases from 47.8359 to 54.3665.

VI. Conclusion Remarks

In this paper, we model the two-settlement system as a two-period game with multiple states of the world in the second period. Because we assume, linear demand functions and constant marginal generation cost the spot market equilibrium can be computed as a linear complementarity problem. In period 1, firms solve an expected utility maximization problem subject to the equality between the forward price and the expected weighted spot prices, and the linear complementarity problem defining the spot market equilibria in period two. This problem is non-convex and generally hard to solve. We introduce two approaches: An iterative PIPA and an iterative RSM. For the example we tested, both approaches generate the same result. We also observe from the example about the likelihood of congestion, generation quantities and social welfare changes due to forward contracts. Our next task is to generalize the cost functions to quadratic curves and perform extensive sensitivity runs on the various parameters.

Our current tests show that the iterative RSM approach is faster than the iterative PIPA approach. Both approaches are sensitive to the number of nodes. However, the iterative PIPA approach is more sensitive to the number of state contingencies while the iterative RSM approach is more sensitive the number of zones and the step size.

Finally, as indicated earlier, we also plan to relax the no-arbitrage assumption between the forward and spot market replacing it with a market clearing condition that sets the forward price based on expected demand functions in the spot market. We expect that such a condition enhances generators market power and will enable generators to raise forward prices above the expected spot prices while increasing their profits.

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Jian Yao is a Ph.D. student in the department of IEOR at the University of California at Berkeley. He was research assistant on project EECOMS (Extended Enterprise coalition for integrated COllaborative Manufacturing Systems) funded by NIST ATP (Advance Technology Program), and software engineer for Advanced Planning & Scheduling products at Oracle Corporation. He has received his M.S. in Computer Science from the Univeristy of North Carolina at Charlotte, and M.S. and B.S in Mechanical Engineering from Shanghai Jiao Tong University.

Dr. Shmuel Oren is Professor of IEOR at the Univ. of California, Berkeley. He is the Berkeley site director of PSERC (the Power System Engineering Research Center). He published numerous articles on aspects of electricity market design and has been a consultant to various private and government organizations including the Brazilian regulatory commission, The Alberta Energy Utility Board the Public Utility Commission, the Polish system operator and to the Public Utility Commission of Texas were he is currently a

Senior Advisor to the Market Oversight Division. He holds a B.Sc. and M.Sc. in Mechanical Engineering and in Materials Engineering from the Technion in Israel and he received an MS. and Ph.D in Engineering Economic Systems in 1972 from Stanford. Dr. Oren is a member of INFORMS and a Fellow of the IEEE.

Dr. Ilan. Adler is a Professor and Chair of the IEOR department at the University of California at Berkeley. His main research activities are in the the area of optimization in general and Interior Point methods in particular. He has also been a consultant to various companies on large, complex optimization problems. He holds a B.A. in Economics and Statistics from the Hebrew University in Israel, M.Sc. in Operations Research from the Technion in Israel and a Ph.D in Operations Research from Stanford. Dr. Adler is a member of INFORMS and the Mathematical Association of America.